M OUT OF N ERROR CORRECTING CODE USING DIGITAL FILTERS

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Abstract:

In this work, forward and reversed digital filters are used with (m out of n) error correcting code in the form [xxxx000] i.e.(4 out of 7code). The hardware circuit is constructed using flip flops that constitute the digital filters. Practical results are applied for correcting a deliberately inserted error in the message.

1- Introduction

There are several known methods of detecting an error in digital message, namely: Redundancy, Echoplex, Exact Count Encoding, Parity, Checksum, Vertical-Horizontal Redundancy Checking, and finally Cyclic Redundancy Checking (CRC).

Most methods use codes which are "systematic": the transmitter sends a fixed number of original data bits, followed by fixed number of check bits (usually referred to as redundancy) which are derived from the data bits by some deterministic algorithm. The receiver applies the same algorithm to the received data bits and compares its output to the received check bits; if the values do not match, an error has occurred at some point during the transmission. A block code converts affixed length of K data bits to a codeword of fixed length N, where N>K. The block of encoded message consists of K message bits in addition to N-K error check bits. The rate of the code is the ratio N/K, and the redundancy of the code is (1-K/N)[1-5].

In this study a correcting code of seven bits is used. The message constitutes the most significant four bits while the rest are zeroes (the added redundancy bits), so the codeword is in the form [xxxx000].

In case of error correction few methods are used namely: Symbol Substitution,

Retransmission, and Forward Error Correction (FEC). Nowadays the plausible method FEC is versatile and applied almost universally. It involves applications of Reed-Soloman coding as indicated in the block diagram of Fig.1, where the first three blocks are common parts for cables, satellites, and terrestrials; while, the last block is used for satellites, and terrestrials only.

In Fig.1, the numbers (204,188,8) stands for Transport Packets of 188 bytes long, it uses 16 parity bytes, when added to 188 yields 204. The 8 means that this method can correct up to 8 bytes of errors. The energy dispersal uses Pseudo Random Binary Sequence (PRBS) of 1503 bytes. Forney interleaving is vital in the sense that it increases the efficiency of Reed-Soloman coding by spreading the burst error in the channel. Convolutional coding (inner coding) is the complement of RS and forney interleaving because it corrects other kinds of errors. It lowers redundancies by the method of puncturing the output of the convolutional encoder.

The technique used in this research is another method of correcting an erroneous byte in a message by the method of "search and compare" the system searches for the correct code in the sense of scanning and then compares the scanned result to a standard (xxxx000) byte. If the scanned result matches this standard code, then the byte involved is the correct message byte.

A hypothetical communication system with artificial erroneous message is considered and a correcting circuit is designed so as to correct the received message.

2- Theoretical Circuit:

2.1- Block Diagram of the system

The theoretical circuit can be represented as shown in Fig.2. The message (x) is passed through a forward digital filter of transfer function $f(D)$at the sending end. During the path of the message towards the receiving end, an error is deliberately inserted in the message using shift and add method (Pseudo Noise [PN] generator)[6,7] . At the receiving end, a reverse digital filter of transfer function $\frac{1}{f(D)}$is used in addition to m out of n facility to detect the error and correct it simultaneously .

2.2-Related Algebra of Digital Filters

The forward digital filter at the sending end is shown at Fig.3 for 3-bits.

Assuming the input and output are $(x)$and (y)respectively, and the delay "by one bit period" to be(D), then[8]

$y = x \oplus Dx \oplus D^3x$----------(1)

from which we get

$f(D) = \frac{y}{x} = 1 \oplus D \oplus D^3$ ----------(2)

Fig.4 shows the reverse digital filter at the receiving end, from which

$y' = x' \oplus D^3y' \oplus Dy'$ ----------(3)

Using transposition property, the following result is obtained:

$x' = y' \oplus Dy' \oplus D^3y'$ ----------(4)

Therefore:

$$\frac{1}{f(D)} = \frac{y'}{x'} = \frac{1}{1 \oplus D \oplus D^3} \quad \text{----------(5)}$$

where (x') and (y') are the input and output respectively and (D) is the delay by one bit period.

3- Circuit Analysis

Fig.5 shows the practical circuit of the detector and corrector system (the scanner) at the receiving end.It is consisted of three registers. The received message is stored in the second register, while the third register is the storage of the corrected message.

The first stage of the system (3-bit shift register) generates the codes due to probability of errors as follows:

Denoting the output of the first register by "z"

i. If the first register of Fig.5 is loaded initially with the logic [100],and the register is drove by clock pulses, the output "z" becomes as in Table_1 .

Table_1: The status of initially loaded shift register of Fig.5 driven by clock pulses

After n^{th} clock pulse	Z	Contents of shift register
Initially	1	100
1	1	110
2	1	111
3	0	011
4	1	101
5	0	010
6	0	001
7	1	100

It is clear from tabl_1 that the final state is the same as the first one, the cycle isthen repeats in such a way that "z" (the output) will have the following recurrence cycles

…..1110100 1110100 1110100…..

ii. Consider the register loaded initially with the logic [110].

If the work in (i) above is repeated, the possible outputs of (z) become

….0111010 0111010 0111010….

iii. If the register is loaded with logic [110] then the possible outputs of (z) are

z =0011101, and so on.

The above functions of [z] are termed ∅E, for which it is clear that the result is shifted one bit to the right each time, so initially the shift register can be loaded by any code, hence the possible outputs of [z] are tabulated as follows:

Table_2 Possible outputs of the first register of Fig.5

Function of [z]	Binary value of [z]
ϕ E_1	1110100
ϕ E_2	0111010
ϕ E_3	0011101
ϕ E_4	1001110
ϕ E_5	0100111
ϕ E_6	1010011
ϕ E_7	1101001

Referring to Fig.2; $x' = y \oplus E$ represents an input to the reverse filter denoted by $\frac{1}{f(D)}$ where E is an error bit inserted to the transmitted message for example the third bit i.e. $E = E_3$.So the output y' of this feedback filter is given by

$y' = \phi(x')$----------(6)

$= \phi[y \oplus E_3]$

$= \phi[y] \oplus \phi[E_3]$

$= \phi f(x) \oplus \phi[E_3]$

$= \phi[D] \oplus x'$

$= \frac{1}{f(D)} \oplus x'$ ----------(7)

y'can also be expressed as

$y' = x \oplus \phi[E_3]$---------- (8)

where x is the input.

From the final results (equations 7&8), it is noticed that the input x is the original message while ϕ [E_3] is the response of the reverse filter to the impulsive error (0010000),hence the output x is expressed as

$x = y' \oplus \phi$ [E_3] ----------(9)

practically the position of the error is not known, hence all of the responses should be examined, namely ϕ[E_1], ϕ[E_2], Ø[E_3],….., ϕ[E_7], (as in Fig.6) which correspond to impulse function that may be at the 1st,2nd,3rd,….,.7th position respectively. Therefore, the general form of equation 9 will be

$x = y' \oplus \phi$ [E_i] ----------(10)

Where ϕ [E_i] is the output generated from the first register of the system.

After achieving all examinations, the XoR operation (according to equation 10) continues until a code with three zeroes at the right hand side (in the form xxxx000) is obtained in the third register, where the correct message is xxxx.

4-Practical Results

The electronic circuit of the detector and corrector system (the scanner) as shown in Figs.2 and 6 has been built practically. An erroneous messagey'=0011011 was chosen as an examplewhich is supposed to be belonging to the seven bit code [xxxx000], then y' is examined according to equation (10), by considering all possible ϕ E_i (from Table_2) until a code of the form xxxx000 is reached as in Table_3:

Table_3: TheXoR results for correcting the erroneous message.

i	ϕ E_i	$y' \oplus \phi E_i$	Result
1	1110100	1101111	Incorrect
2	0111010	0100001	Incorrect
3	0011101	0000110	Incorrect
4	1001110	1010101	Incorrect
5	0100111	0111100	Incorrect
6	1010011	1001000	Correct
7	1101001	1110010	Incorrect

Hence, the error is at the 6th position of the message and the message is corrected.

5-Conclusions:

This research work claims juniority and originality, whereas in the existed literature the standard methodof error detection and correction (as mentioned above), the plausible of which is the FEC that utilizes either Hamming code or Reed-Soloman code, none of such methods includes “SCAN and COMPARE” method as in this work.

A standard (N,K) code, in this research, i.e. (xxxx000 code) was applied as a stem or a reference to all received messages. Such received messages are either correct or faulty. If erroneous message is received it is corrected so as to match the reference (xxxx000) code..

A final word can be added that to comprehend the literature of this research the reader may concentrate on sequential technique given in the tables throughout this work, the picture then will be elaborated and the ambiguousness will be cleared, consequently the practicality of the circuit will be realized and hence anticipated.

This piece of work can be an integral part of a system that involves digital communication based on this (N,K) code, or else may be developed to further usage and extensions yielding same trend or same principles.

6- References:

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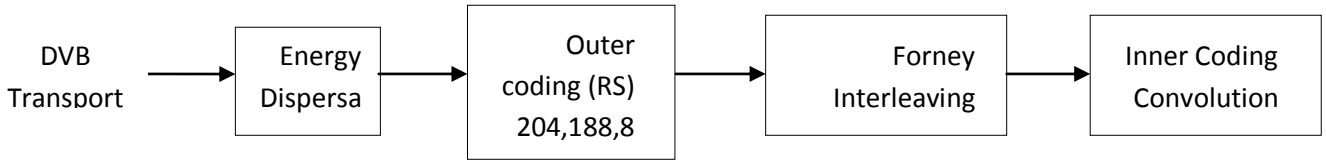

Fig.1- Block diagram of FEC system

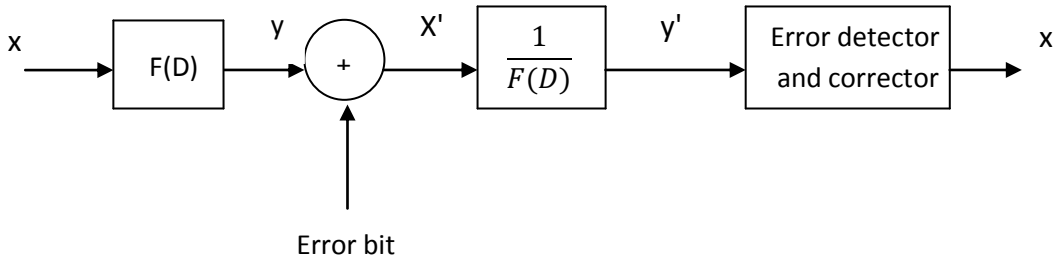

Fig.2_Theoretical circuit

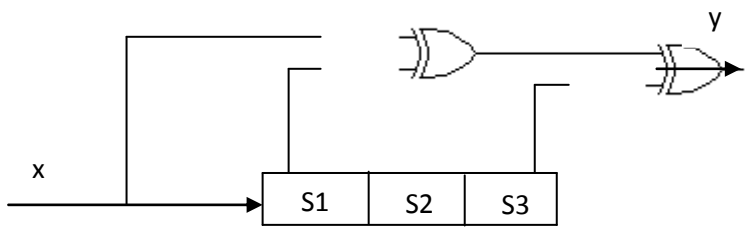

Fig.3_ Forward digital filter

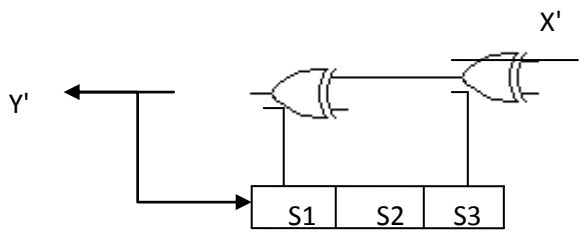

Fig.4_Reversed digital filter

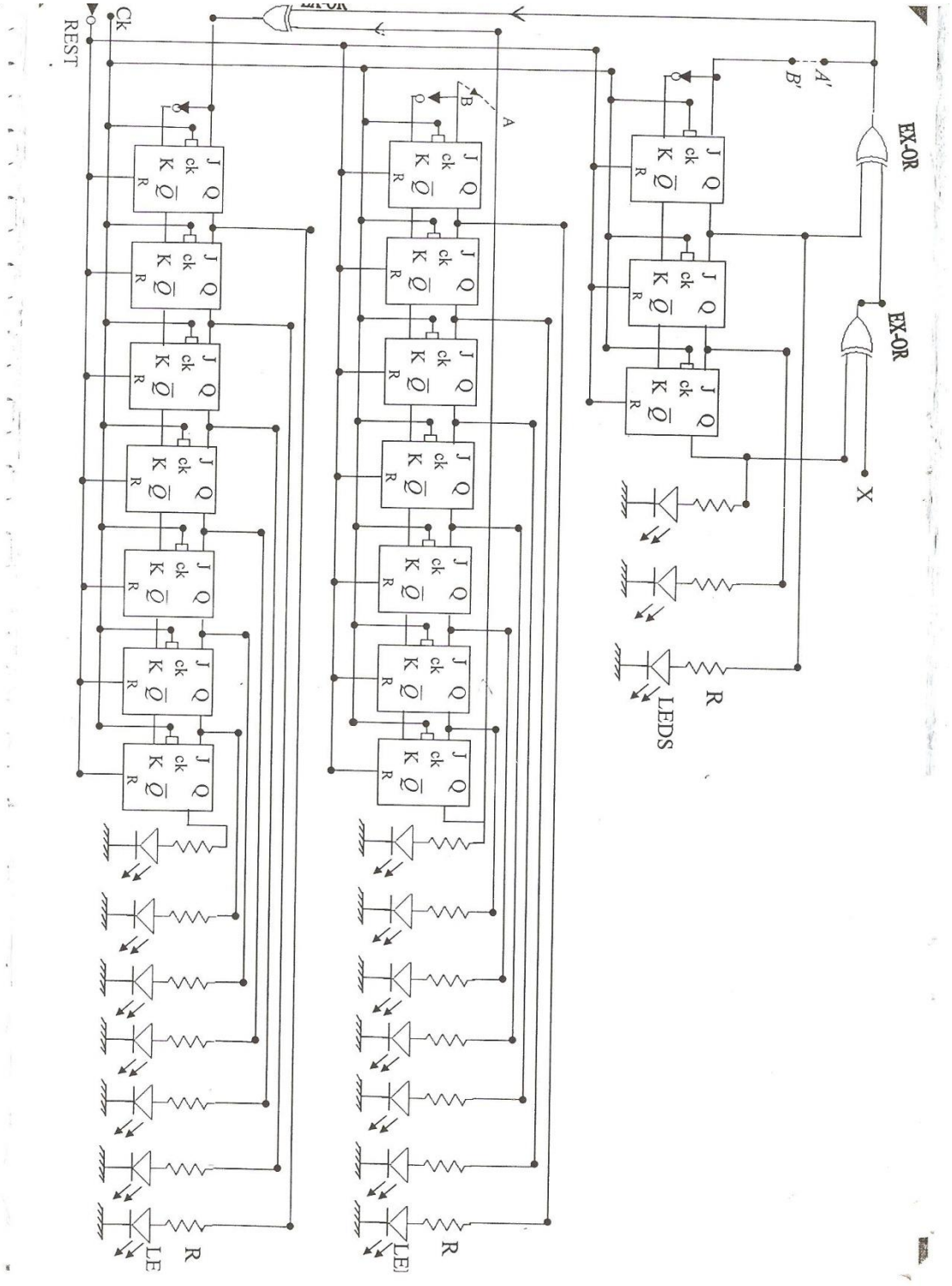

Fig .5 Practical circuit

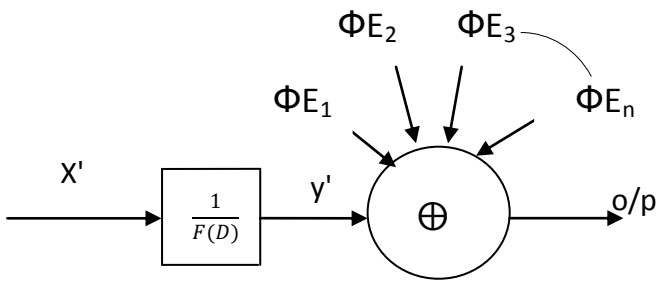

Fig.6 Theoretical circuit of the receiving end

شفرة m من أصل n لتصحيح الخطأ باستخدام المرشحات الرقمية

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المستخلص

يتعلق العمل باستخدام مرشحات رقمية أمامية وعكسية مع شفرة (m out of n) لتصحيح الخطأ والتي تكون بالصيغة (xxxx000)، (أي شفرة 4 من أصل 7) حيث يتم استخدام نطاطات لتمثيل المرشحات الرقمية في بناء الدائرة. ولقد تم عرض النتائج العملية لتصحيح خطأ محقق في إشارة المعلومات المرسلة عمدا.